ELECTROSTATICS

Coulomb force between two point charges

$$\vec{F} = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q_1q_2}{|\vec{r}|^3}\vec{r} = = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q_1q_2}{|\vec{r}|^2}\hat{r}$$

- **The electric field intensity at any point is the force experienced by unit positive charge, given by** $\vec{E} = \frac{\vec{F}}{q_0}$
- **Electric force on a charge 'q' at the position of electric field intensity** $\vec{E}$ **produced by some source charges is** $\vec{F} = q\vec{E}$
- **Electric Potential**

If $(W_{\infty P})_{ext}$ is the work required in moving a point charge q from infinity to a point P, the electric potential of the point P is

$$V_p = \left.\frac{(W_{\infty p})_{ext}}{q}\right]_{acc=0}$$

- **Potential Difference between two points A and B is**

  $V_A - V_B$

- **Formulae of $\vec{E}$ and potential V**

(i) Point charge $E = \frac{Kq}{|\vec{r}|^2}\cdot\hat{r} = \frac{Kq}{r^3}\vec{r}$, $V = \frac{Kq}{r}$

(ii) Infinitely long line charge $\frac{\lambda}{2\pi\varepsilon_0 r}\hat{r} = \frac{2K\lambda\hat{r}}{r}$

V = not defined, $v_B - v_A = -2K\lambda \ \ln(r_B / r_A)$

(iii) Infinite nonconducting thin sheet $\frac{\sigma}{2\varepsilon_0}\hat{n}$,

V = not defined, $v_B - v_A = -\frac{\sigma}{2\varepsilon_0}(r_B - r_A)$

(iv) Uniformly charged ring

$E_{axis} = \frac{KQx}{(R^2 + x^2)^{3/2}}$, $E_{centre} = 0$

$V_{axis} = \frac{KQ}{\sqrt{R^2 + x^2}}$, $V_{centre} = \frac{KQ}{R}$

x is the distance from centre along axis.

(v) Infinitely large charged conducting sheet $\frac{\sigma}{\varepsilon_0}\hat{n}$

V = not defined, $v_B - v_A = -\frac{\sigma}{\varepsilon_0}(r_B - r_A)$

(vi) Uniformly charged hollow conducting/ nonconducting /solid conducting sphere

(a) for $\vec{E} = \frac{kQ}{|\vec{r}|^2}\hat{r}$, $r \geq R$, $V = \frac{KQ}{r}$

(b) $\vec{E} = 0$ for r < R, $V = \frac{KQ}{R}$

(vii) Uniformly charged solid nonconducting sphere (insulating material)

(a) $\vec{E} = \frac{kQ}{|\vec{r}|^2}\hat{r}$ for $r \geq R$, $V = \frac{KQ}{r}$

(b) $\vec{E} = \frac{KQ\vec{r}}{R^3} = \frac{\rho\vec{r}}{3\varepsilon_0}$ for $r \leq R$, $V = \frac{\rho}{6\varepsilon_0}(3R^2 - r^2)$

(viii) thin uniformly charged disc (surface charge density is σ)

$$E_{axis} = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{R^2 + x^2}}\right] \qquad V_{axis} = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{R^2 + x^2} - x\right]$$

- **Work done by external agent in taking a charge q from A to B is** $(W_{ext})_{AB} = q\,(V_B - V_A)$ or $(W_{el})_{AB} = q\,(V_A - V_B)$.

- **The electrostatic potential energy of a point charge** $U = qV$

- **U = PE of the system =**

$$\frac{U_1 + U_2 + ...}{2} = (U_{12} + U_{13} + ... + U_{1n}) + (U_{23} + U_{24} + ... + U_{2n}) + (U_{34} + U_{35} + ... + U_{3n}) ....$$

- **Energy Density** = $\frac{1}{2}\varepsilon E^2$

- **Self Energy of a uniformly charged shell** = $U_{self} = \frac{KQ^2}{2R}$
- **Self Energy of a uniformly charged solid non-conducting sphere**

$$= U_{self} = \frac{3KQ^2}{5R}$$

- **Electric Field Intensity Due to Dipole**

(i) on the axis $\vec{E} = \frac{2K\vec{P}}{r^3}$

(ii) on the equatorial position : $\vec{E} = -\frac{K\vec{P}}{r^3}$

(iii) Total electric field at general point O (r,θ) is $E_{res} = \frac{KP}{r^3}\sqrt{1 + 3\cos^2\theta}$

- **Potential Energy of an Electric Dipole in External Electric Field:**

$$U = -\vec{p}.\vec{E}$$

- **Electric Dipole in Uniform Electric Field :**

$$\text{torque } \vec{\tau} = \vec{p} \times \vec{E}\,; \quad \vec{F} = 0$$

- **Electric Dipole in Nonuniform Electric Field:**

$$\text{torque } \vec{\tau} = \vec{p} \times \vec{E}\,;\ U = -\vec{p}\cdot\vec{E}\,,\ \text{Net force } |F| = \left|p\frac{\partial E}{\partial r}\right|$$

- **Electric Potential Due to Dipole at General Point (r, θ) :**

$$V = \frac{P\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{\vec{p}\,.\,\vec{r}}{4\pi\varepsilon_0 r^3}$$

- **The electric flux over the whole area is given by**

$$\phi_E = \int_S \vec{E}.\overrightarrow{dS} = \int_S E_n dS$$

- **Flux using Gauss's law,** Flux through a closed surface

$$\phi_E = \oint \vec{E}\cdot\overrightarrow{dS} = \frac{q_{in}}{\varepsilon_0}.$$

- **Electric field intensity near the conducting surface**

$$= \frac{\sigma}{\varepsilon_0}\hat{n}$$

- **Electric pressure :** Electric pressure at the surface of a conductor is given by formula

$$P = \frac{\sigma^2}{2\varepsilon_0}$$ where σ is the local surface charge density.

- Potential difference between points A and B

$$V_B - V_A = -\int_A^B \vec{E}.d\vec{r}$$

$$\vec{E} = -\left[\hat{i}\frac{\partial}{\partial x}V + \hat{j}\frac{\partial}{\partial x}V + \hat{k}\frac{\partial}{\partial z}V\right] = -\left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial x} + \hat{k}\frac{\partial}{\partial z}\right]V$$

$$= -\nabla V = -\text{grad } V$$